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On mechanical response in a piezoelectric transducer carrying a time-decaying space-charge

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The transform calculus is used to determine the mechanical response in a piezoelectric transducer carrying a time-decaying space-charge. The response is found to be partly linear, partly constant and partly transient in character to a first order of approximation.

1. Introduction

The studies on piezoelectric transducers are very important in view of their applications in manifold branches of physics, engineering and technology, particularly in ultrasonics, for they provide the methods for detecting ultrasonic waves, vide, Redwood (1961a). The piezoelectric materials used for ultrasonic purposes can emit mechanical responses due to electrical excitations and vice-versa. Mason (1948) has carried out this analysis by pursuing the principles of circuit theory. The methods of continuous media were introduced by Redwood (1961a), (1968b) and some papers have been contributed by Sinha (1965), (1967a), (1963), (1967b), Giri (1966), Roy (1967), Das (1967). The author here attempts to determine the mechanical response in a piezoelectric transducer carrying a time-decaying space-charge. The response is found to be partly transient, partly constant and partly linear in character to a first order of approximation.

2. Fundamental equations and boundary conditions

We consider a piezoelectric transducer in the form of a bar executing vibration in the thickness direction which we take to be the x-axis. To obtain the mechanical response we are to couple the equations connecting the two fields mechanical and electrical.

The piezoelectric equations of state in one-dimensional strain is

$$T = cS - hD \qquad \text{...(1)}$$

$$E = -hS + \frac{D}{\epsilon} \qquad \text{...(2)}$$

where T is the stress, S the strain, E the electric field strength, D the electric displacement, c the elastic stiffness for constant E, h the piezo-electric constant measured with D constant and ϵ the permittivity for constant S.

From Newton's second law, $\rho \frac{\delta^2 \xi}{\delta t^2} = \frac{\delta T}{\delta x} = c \frac{\delta^2 \xi}{\delta x^2} - h \frac{\delta D}{\delta x}$ ...(3)

by (1), where ρ is the density of the material, ξ the mechanical displacement.

We assume the variation of space-charge ρ_1 with time in the form given by $\rho_1 = \rho_0 e^{-\alpha t}$, α (> 0), ρ_0 are constants.

so that we can write $\frac{\delta D}{\delta x} = \rho_0 e^{-\alpha t}$. ...(4)

Then (3) becomes $\rho \frac{\delta^2 \xi}{\delta t^2} = c \frac{\delta^2 \xi}{\delta x^2} - h\rho_0 e^{-\alpha t}$.

The Laplace's transform of this equation gives

$$\frac{\delta^2 \bar{\xi}}{\delta x^2} - \frac{\rho p^2 \bar{\xi}}{c} = \frac{h\rho_0}{c(p+\alpha)}$$

The solution of this equation gives

$$\bar{\xi} = Ae^{-\frac{px}{v}} + Be^{\frac{px}{v}} - \frac{h\rho_0}{\rho p^2(p+\alpha)} \qquad \text{...(5)}$$

where $v^2 = \frac{c}{\rho}$, A and B are amplitude factors to be determined from the boundary conditions.

Also from the equation (4), $D = x\rho_0 e^{-\alpha t}$, D being assumed to be zero at $x = 0$.

Taking Laplace's transform, we have $\bar{D} = \frac{x\rho_0}{p+\alpha}$. ...(6)

From (1) we get

$$\bar{T}yz = cYZ\left(\frac{\delta\xi}{\delta x}\right) - h\bar{D}YZ$$

$$\therefore\ \bar{F} = cYZ\frac{\delta\xi}{\delta x} - \frac{hx\rho_0 YZ}{p+\alpha} \qquad \ldots(7)$$

where F is the force exerted over an area L normal to x —axis.

The boundary equations are given by the conditions of continiuty of force and displacement at its two extremities where two mechanical systems may be attached. Let the entities of the corresponding systems at the two extermities $x = o$ and $x = X$ be denoted by the suffixes 1 and 2 respectively. Thus we have

$$(\xi)_o = (\xi_1)_o \qquad \ldots(8)$$

$$(\bar{F})_o = (\bar{F}_1)_o \qquad \ldots(9)$$

$$(\bar{F})_X = 0 \qquad \ldots(10)$$

3. Solution of the problem

To achieve the solution of the problem, in a general way we associate with the transducer with two mechanical systems. We assume the disturbance to proceed from the system 1, and assume the transducer to be rigidly backed at $x=X$.

These give from (5) and (8),

$$A + B = B_1 \qquad \ldots(11)$$

From (7) and (9),

$$v(-A + B) = v_1 B_1 \qquad \ldots(12)$$

From (10),

$$-\frac{hX\rho_0 YZ}{p+\alpha} + \rho v YZp\left(-A\ {}^{\frac{p}{v}X} + B\,e^{\frac{p}{v}X}\right) = 0 \qquad \ldots(13)$$

Eliminating B_1 from (11), (12) and (13) we get

$$A = \frac{hX\rho_0(v-v_1)}{\rho p v(p+\alpha)\left\{(v+v_1)\,e^{\frac{p}{v}X} - (v-v_1)\,e^{\frac{p}{v}X}\right\}}$$

$$B = \frac{hX\rho_0(v+v_1)}{\rho p v(p+\alpha)\left\{(v+v_1)\,e^{\frac{p}{v}X} - (v-v_1)\,e^{\frac{p}{v}X}\right\}}$$

$$(\bar{\xi})_0 = \frac{hX\rho_0}{\rho p v(p+\alpha)}\left\{\frac{2v}{(v+v_1)e^{\frac{p}{v}X}-(v-v_1)e^{\frac{p}{v}X}}\right\} - \frac{h\rho_0}{\rho p^2(p+\alpha)}$$

$$= \frac{2hX\rho_0}{\rho p(p+\alpha)}\,\frac{e^{\frac{p}{v}X}}{v+v_1}\left\{1-\frac{v-v_1}{v+v_1}e^{\frac{2p}{v}X}\right\}^{-1} - \frac{h\rho_0}{\rho p^2(p+\alpha)}$$

The inversion of it being too complicated, we proceed as in Redwood (1961), to obtain it for small values of time, and expanding as in Redwood (1961), we get

$$(\bar{\xi})_0 = \frac{2hX\rho_0\, e^{\frac{p}{v}X}}{\rho p(v+v_1)\,(p+\alpha)} - \frac{h\rho_0}{\rho p^2(p+\alpha)} = \frac{\theta_1 e^{\frac{p}{v}X}}{p(p+\alpha)} + \frac{\theta_2}{p^2} + \frac{\theta_3}{p} + \frac{\theta_4}{p+\alpha}$$

where θ_1, θ_2, θ_3, θ_4 are constants containing material parameters of the problem.

Taking inverse transform, $\xi = \theta_3 + \theta_2 t + (\theta_4\, e^{-\alpha t}$

$$+\frac{\theta_2}{\alpha}\left\{1-e^{-\alpha t}\right\} u\left(t-\frac{x}{v}\right) \text{ where } t > \frac{X}{v}$$

$$\text{where } u\left(t-\frac{x}{v}\right) = 0 \text{ for } t < \frac{x}{v}$$

$$= 1 \text{ for } t > \frac{x}{v}$$

This shows that the mechanical response in a piezoelectric transducer carrying time-decaying space-charge is partly linear, partly constant and partly transient in character.

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